



# 以游戏为例:概率与统计的应用与探索

Exploration and Application: Probability and Statistical  
Theory in Games

### #1. 课程背景及简介



本课题第一个需要理解的概念是概率，是适用于随机事件的。这是一种用数学方法衡量某件事发生或不发生的可能性的方法。首先要知道的是概率总是一个介于 0 和 1 之间的数。你需要理解一个永远不会发生的事件的概率是 0。一个总是发生的事件的概率是 1。任何可能发生或不可能发生的事情都可以用 0 到 1 之间的数字来衡量，这个数字很容易计算。教授将带领大家探索这个神秘的由数字组成的世界，是如何与现实中的息息相关的。本课题旨在让学生注意到数学在许多人感兴趣的领域上的应用。希望学生们反过来会对背后的数学感兴趣，并看到数学是如何在现实世界中产生影响的。会以游戏为例，解释概率论中的许多概念将在扑克游戏的应用中进行介绍。将展示庄家如何始终可以处于游戏中的优势，这将在数学上得到证明。总的来说，许多实验表明，人类大脑通常在处理概率和机会方面存在严重问题。对概率的更深入理解有助于培养应对风险所需的直觉，从而能够做出更明智的决策，从而做出更好的决策。

### #2. 学习目标



本课程将解决许多挑战，如：

- ★ 概率学基本规律
- ★ 轮盘与筛子组成规律
- ★ 计数和组合以及统计学、二项式系数
- ★ 博弈理论（囚徒困境等）

### #3. 任课教师信息



**Prof. I. A.**  
苏黎世联邦理工学院数学系正教授，美国新泽西州普林斯顿高等研究院成员 (Institute for Advanced Study in Princeton, NJ, USA)，英国剑桥艾萨克·牛顿研究所成员，在意大利 Università di Roma La Sapienza 完成了数学学习，并获得了 Laura(最高荣誉)。

### #4. 课程设置



周期	时间	课程设置内容	课时
第一周 学习指南 教授及助	1 月 28 日 周六	什么是 PBL 教学方法	1
		PBL 教学的常见形式	1
	1 月 29 日	教授课-1	3



教辅导	周日	交叉学科 PBL 课程设计及知识点学习 学习目标： 概率和统计概论及其区别 概率的定义和性质 轮盘	
	1 月 30 日 周一	助教课-1 知识点查漏补缺	2
	1 月 31 日 周二	教授课-2 制定小组项目方向 学习目标： 扑克和德州扑克 随机变量和期望值 阶乘、排列和组合	3
第二周 教授及助 教辅导	2 月 1 日 周三	助教课-2 知识点查漏补缺	2
	2 月 2 日 周四	教授课-3 交叉学科课程知识点学习 学习目标： 骰子：二项分布和正态分布 方差和标准差。 条件期望、贝叶斯公式	3
	2 月 3 日 周五	助教课-3 知识点查漏补缺& 跟进学生个人作业进度	2
	2 月 4 日 周六	教授课-4 互动与项目设计跟进答疑	1.5
	2 月 6 日 周一	助教课-4 跟进学生个人作业进度	2
	2 月 7 日 周二	教授课-5 交叉学科课程知识点学习 学习目标： 博弈论基础：零和博弈、优势和鞍点、混合策略	2
	2 月 8 日 周三	助教课-5 跟进学生个人作业进度	2
	2 月 9 日 周四	教授课-6 交叉学科课程知识点学习 学习目标： 非零和博弈、运动图、纳什均衡、帕累托最优性	2

第三周 教授及助教辅导 未来展望	2月10日 周五	助教课-6 知识点查漏补缺& 指导学生个人作业成果展示	2
	2月11日 周六	教授课-7 教授点评小学生个人作业成果	1.5
	2月12日 周日	升学与就业方向展望	1
		个人规划及发展建议	1
总课时	32		

#5. 阅读材料



- ★ David G. Taylor, “Games, gambling and probability”, 2021, CRC Press
- ★ Ronald J. Gould, “Mathematics in Games, Sports and Gambling”, 2010, CRC Press
- ★ Edward W. Packel, “The Mathematics of Games and Gambling”, 2006, The Mathematics Association of America
- ★ Martin J. Osborne, “An introduction to Game Theory”, 2000, Oxford University Press
- ★ Ashok P. Maitra and William D. Sudderth, “Discrete Gambling and Stochastic Games”, 1996, Springer-Verlag

#6. 项目主题



本课程使用 PBL 教学法，PBL 即项目式学习，是一种以学生为中心的教学方法，教师提供关键素材构建学习环境，学生通过在此环境里解决一个开放式项目的经历来学习。以下为本课程可选的项目主题：

- 1. 真正的随机性？（[1] 第 11.1 条）
- 2. 循环计算“斐波那契”硬币（[1] 的第 11.3 节）
- 3. 组成和概率（[1] 的第 11.4 节）
- 4. Sicherman 骰子（[1] 第 11.5 节）
- 5. 旅行推销员（[1] 第 11.6 节）
- 6. 随机游走和生成函数（[1] 的第 11.7 节）
- 7. 圣彼得堡悖论（[1] 的附录 B）
- 8. 二项分布与正态分布（[1] 的附录 C）

学生应该理解上述所选的主题并写下自己的观点。

英文版教学大纲





Course Title	Exploration and Application: Probability and Statistical Theory in Games
Credit Hours	32 (one credit hour is 45 minutes)
Course Objectives	<p>This class will address challenges such as:</p> <ul style="list-style-type: none"><li>★Basic laws of probability</li><li>★The composition rule of roulette and sieve</li><li>★Counting and Combining and Statistics, Binomial Coefficients</li><li>★Game Theory (Prisoner's Dilemma, etc.)</li></ul>
Course Description	<p>Many of the concepts in probability theory will be introduced with the help of games such as Roulette, Poker, Liar's Dice. This is because historically probability stemmed from Girolamo Cardano first (unpublished for many years), then later with Chevalier de Méré with Blaise Pascal and Pierre de Fermat trying to understand whether the house or the player had the advantage in a variation of the game according to which the payer rolls four dice and wins provided none of them is a six. We will go very quickly over the basic concepts of probability (if you are not familiar with them you can read the first two chapters of [1]). We will look at the Law of Large Numbers, counting, a bit of combinatorics, distributions. The final aim of the course is to develop some understanding of game theory, starting from bluffing and continuing with non-zero- sum games, Pareto Optimality and the Prisoner's Dilemma. As many of the examples throughout the course will be games, at the end of the course it should be clear why betting in a casino is a bad idea! In fact it will be shown how the house has always the advantage over a player, and this will be proven mathematically.</p>





**Brief introduction of the course**

The course will take place in five sessions of two hours each. We will start with a description of the difference between statistics and probability, followed by a brief introduction of both topics and an illustration of how they both have a place in most of aspects of everyday life. The focus of the course will be mostly on probability theory, although concepts of statistics will be introduced. The course will be completed by a treatment of game theory, another related branch of mathematics that provides tools to analyze situations in which the parties, called players, make decisions that are interdependent. Game theory has also applications in many diverse aspects, including social sciences, philosophy, product pricing and many more.

	Topics
Module 1	Objective: Introduction to probability and statistics, and their difference. Definition and properties of probability. Roulette.
Module 2	Objective: Poker and Texas Hold'Em. Random variables and expected value. Factorial, permutations and combinations.
Module 3	Objective: Liar's dice: Binomial distribution and normal distributions.Mean, variance and standard deviation. Conditional expectation,Bayes' formula.
Module 4	Objective: Basics of game theory: zero sum games, dominance and saddle points, mixed strategies
Module 5	Objective: Non-zero sum games, movement diagrams, Nash equilibrium, Pareto optimality.
Module 6	Objective: Project Discussion Description: a live session to discuss students' projects
Module 7	Objective: Project Presentation Description: a live session where students present their projects

**Required Readings**

- ★ David G. Taylor, "Games, gambling and probability", 2021, CRC Press
- ★ Ronald J. Gould, "Mathematics in Games, Sports and Gambling", 2010, CRC Press
- ★ Edward W. Packel, "The Mathematics of Games and Gambling", 2006, The Mathematics Association of America



★ Martin J. Osborne, "An introduction to Game Theory", 2000, Oxford University Press

★ Ashok P. Maitra and William D. Sudderth, "Discrete Gambling and Stochastic Games", 1996, Springer-Verlag

### **Suggested list of the topics for the final project**

1. True randomness? (§ 11.1 of [1])
  2. Counting "Fibonacci" coins circularly (§ 11.3 of [1])
  3. Compositions and probabilities (§ 11.4 of [1])
  4. Sicherman dice (§ 11.5 of [1])
  5. Traveling salesmen (§ 11.6 of [1])
  6. Random walks and generating functions (§ 11.7 of [1])
  7. St. Petersburg's paradox (Appendix B of [1])
  8. Binomial distribution versus normal distribution (Appendix C of [1])
- The students should understand the selected topic and write an account of it. In some cases (such as topic 3, for example) it will be necessary to read other parts of the book.

### **Criteria**

Participation

Final exam

### **Class Expectation**

Besides being able to apply probability games, you will be able to understand also how it plays an important role in other aspects of life. Finally, an understanding of probability and the related statistics, will certainly give you a head start in any job which is not only related to literature.